

**Comments on Lange,
*Because without Cause***

Kenny Easwaran

Outline of book

- Part I: “distinctively mathematical” explanations by constraint within the sciences.
- Part II: other non-causal explanations - “really statistical” and “dimensional”
- Part III: explanations within mathematics
- Part IV: how do all these explanations go together?

“I do not claim there is an illuminating core essence of explanation — a single overarching explanatory schema ... Nevertheless it is no mystery why various important elements of mathematical and scientific practice are all alike termed ‘explanations’.” (p. 361)

My attempt at an “illuminating core essence”

- To explain a fact is to show that the fact is modally robust.
- The explanans must have sufficient robustness to account for the full modal robustness of the explanandum.
 - “Modal robustness” can be necessity, probability, or symmetry.
- The explanans must account for what is salient in the explanandum.

Explanation of laws

- Newton's First Law: In the absence of forces, a body at rest will stay at rest, and a body in motion will stay in that same motion.
- Newton's Second Law: $F=ma$.

Hierarchy of necessity

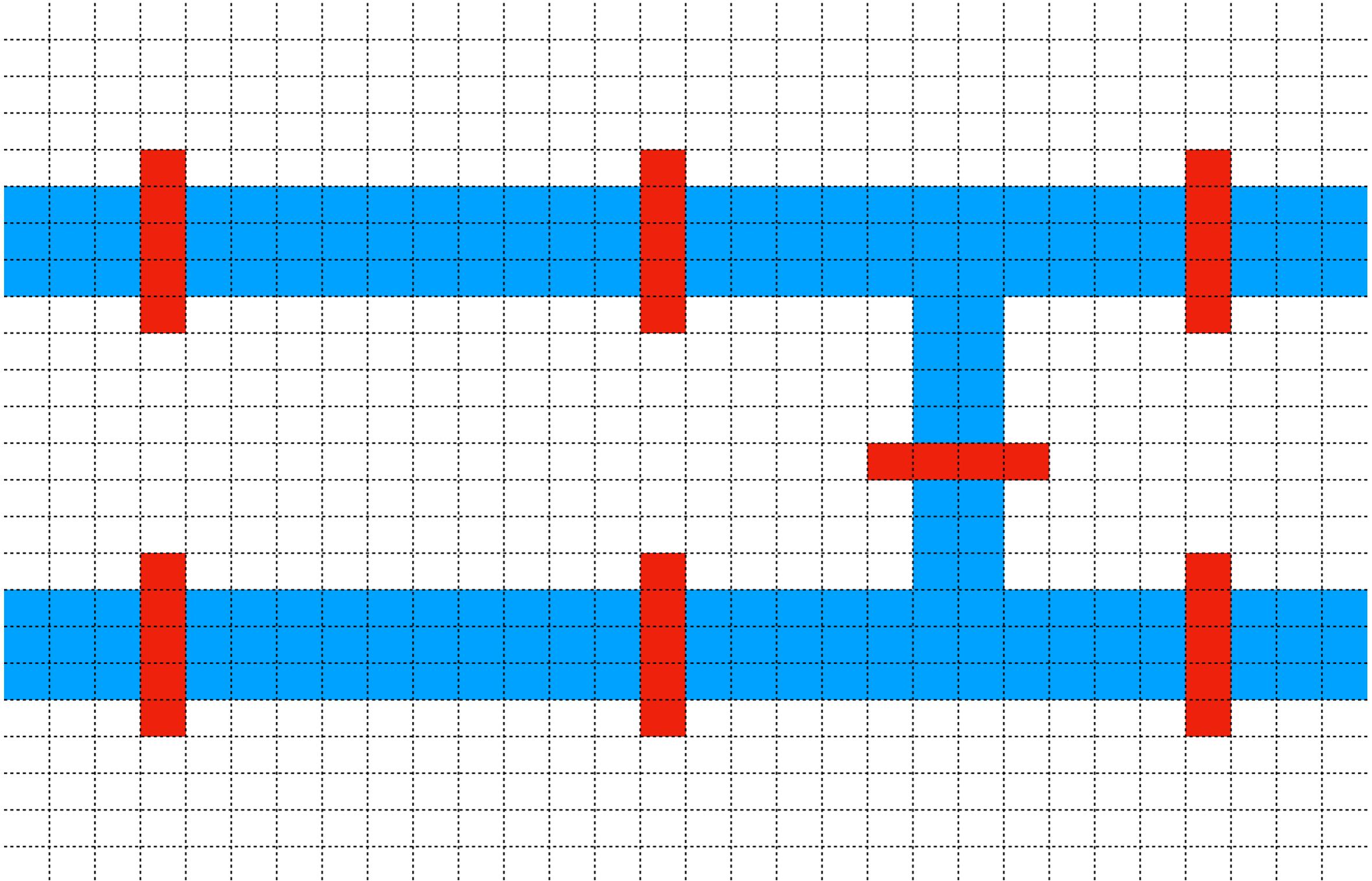
- Gravity is given by GMm/r^2 ; electrostatic force is given by Cq_1q_2/r^2
- Forces are mediated by fields propagating through 3-dimensional space
- Objects interact by forces that determine the second derivative of position
- The laws are the same from any inertial frame of reference

“Subnomic Stability”

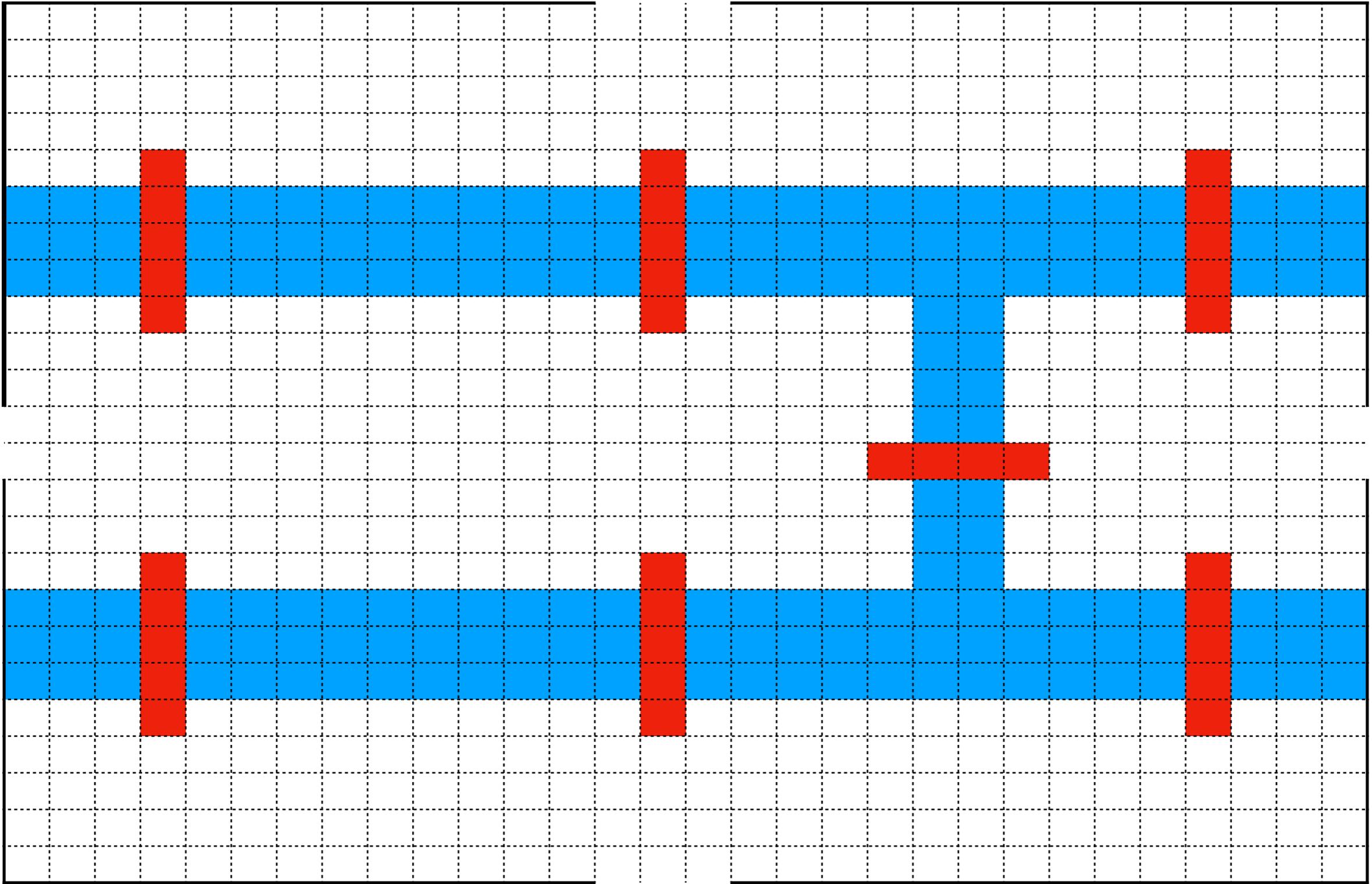
- A “subnomic” statement is one that doesn’t use the concept of a law.
- Let G be a set of true, subnomic sentences closed under logical consequence. G is “subnominally stable” iff for any p that is consistent with the set, and m that is a member of it, the sentence “if p had been true then m would still be true” is true in all conversational contexts.

Subnomically stable sets

- The set of all truths is subnomically stable
- The set of all laws is subnomically stable
- For any two subnomically stable sets, one is a subset of the other. (“If $\sim(S_1 \& S_2)$ had been true, then S_1 would still have been true”; “If $\sim(S_1 \& S_2)$ had been true, then S_2 would still have been true”)
 - This proof only goes through if the relevant counterfactuals are true in the *same* conversational contexts.



“the order of causal priority is not responsible for the order of explanatory priority in distinctively mathematical scientific explanations. Rather, the facts doing the explaining are eligible to explain by virtue of being modally more necessary than ordinary laws of nature (as both mathematical facts and Newton’s second law are) or being understood in the why question’s context as constitutive of the physical task or arrangement at issue.” (p. 33)



Zeitz's coin

- A coin is chosen with a bias somewhere in $[0,1]$ uniformly at random, and then flipped 2000 times. What is the probability that the coin comes up heads exactly 1000 times?
- Answer: $1/2001$
- Non-explanatory proof: $\int_0^1 \binom{2000}{1000} p^{1000} (1-p)^{1000} dp = 1/2001$
- Explanatory proof: We could realize each coin flip by choosing a number in $[0,1]$ uniformly at random and saying that the coin comes up heads iff the new number is below the number chosen for the bias. The number of tails we get is then the rank of the bias among the 2001 numbers chosen. Since each number was chosen with the same distribution, the probability of each is equal. In particular, the probability of 1000 heads is exactly $1/2001$.

Zeitz's coin

- A coin is chosen with a bias somewhere in $[0,1]$ uniformly at random, and then flipped 2000 times. What is the probability that the coin comes up heads exactly k times?
- Answer: $1/2001$
- Non-explanatory proof: $\int_0^1 \binom{2000}{k} p^k (1-p)^{2000-k} dp = 1/2001$
- Explanatory proof: We could realize each coin flip by choosing a number in $[0,1]$ uniformly at random and saying that the coin comes up heads iff the new number is below the number chosen for the bias. The number of tails we get is then the rank of the bias among the 2001 numbers chosen. Since each number was chosen with the same distribution, the probability of each is equal. In particular, the probability of k heads is exactly $1/2001$.

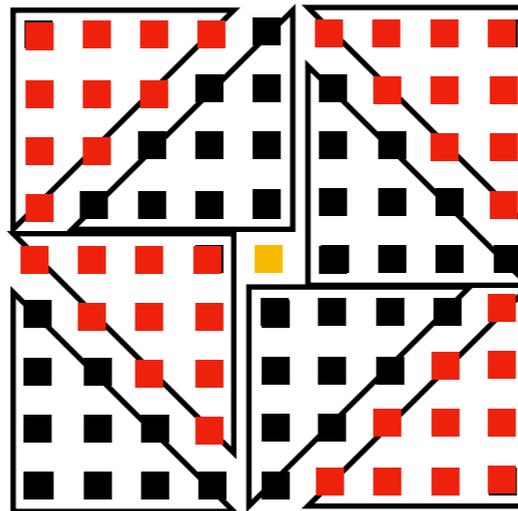
If T is a triangular number, then $8T+1$ is a square.

$$8 \cdot 1 + 1 = 9$$

$$8 \cdot 3 + 1 = 25$$

$$8 \cdot 6 + 1 = 49$$

$$8 \cdot 10 + 1 = 81$$



- Can all explanation be seen as subsumption under some “modally” robust generalization?
- If there are restricted contexts rather than universal contexts in the subnomic stability formulation, then can there be multiple distinct hierarchies of necessity?
- Are there mixed causal and non-causal explanations or can the causal part always be taken as constitutive of the explanandum?
- Can explanation within mathematics be more closely unified with the others?